

# A Simple but Powerful Theory of the Moon Illusion

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Modification of Restle's theory (1970) explains the moon illusion and related phenomena on the basis of three principles: (1) The apparent sizes of objects are their perceived visual angles. (2) The apparent size of the moon is determined by the ratio of the angular extent of the moon relative to the extents subtended by objects composing the surrounding context, such as the sky and things on the ground. (3) The visual extents subtended by common objects of a constant physical size decrease systematically with increasing distance from the observer. Further development of this theory requires specification of both the components of the surrounding context and their relative importance in determining the apparent size and distance of the moon.

The moon at the horizon appears exceptionally large in relation to its apparent size at the zenith. The change in apparent size ranges between 15% and 100% for different observers—a 30% difference is common (Rock & Kaufman, 1962). There has been renewed interest in this problem, and recently, an entire book has been devoted to the controversies surrounding the illusion (Hershenson, 1989). Despite this intense effort, no theory can account for all the relevant facts and observations, and most theories have not been empirically tested. Here, we offer a variant of Restle's theory (1970) of the illusion, which is also supported by well-established principles of perception and empirical results (Baird, 1982; Baird & Wagner, 1982; Wagner, Baird, & Barberesi, 1981). Although simple in formulation, the theory is surprisingly good at resolving the major questions associated with the moon illusion.

First, several primary facts should be explained. (1) As the moon rises, its apparent size systematically decreases (Holway & Boring, 1940). (2) As the moon rises, its apparent distance increases; that is, the smaller the apparent size of the moon, the greater its apparent distance (McCready, 1986). (3) If the visual terrain is eliminated, as can be achieved by looking through a tube, the illusion disappears and the apparent size and distance no longer vary with elevation (Kaufman & Rock, 1962). (4) If the visual terrain is present, the greater the perceived distance to the horizon, the greater the apparent size of the moon (Kaufman & Rock).

In addition to these agreed-upon facts, a number of subsidiary observations have been made about the illusion (Kaufman & Rock, 1962; Rock & Kaufman, 1962). (1) The apparent size of the moon at the horizon is enhanced when the moon is framed by buildings. (2) The apparent size of the moon at the horizon is enhanced by the presence of adjacent clouds. (3) The illusion is reduced somewhat by inverting the view of the terrain, accomplished either by observing the

moon with the head inverted or by viewing an optically inverted scene.

## Previous Theories

Numerous theories have been proposed to account for the moon illusion. The most influential one was proposed by Rock and Kaufman (1962) and Kaufman and Rock (1962). They rely on a perceptual version of the geometry that defines the relation between an object's physical size, its physical distance from an observer, and its subtended visual angle at the eye. That is, geometry dictates that

$$\theta = \tan^{-1}\left(\frac{s}{d}\right), \quad (1)$$

where  $\theta$  is the visual angle,  $s$  is the metric size, and  $d$  is the metric distance. Simplifying<sup>1</sup> and substituting perceived (primed values) for physical values (cf. McCready, 1985),

$$\theta' = \frac{s'}{d'}, \quad (2)$$

where perceived size ( $s'$ ) and perceived distance ( $d'$ ) refer to an observer's estimate of metric extents and perceived visual angle ( $\theta'$ ) refers to an observer's estimate of the proportion of the visual field (the angular extent) occupied by an object.

According to Rock and Kaufman's (1962) argument (though not presented by them as a mathematical model), the perceived size of the moon can be represented by rearranging Equation 2:

$$s' = d'\theta'. \quad (3)$$

Then, by assuming  $\theta'_h = \theta'_z$  and  $d'_h > d'_z$ , we can conclude that  $s'_h > s'_z$  (Fact 1).

In order to handle the fact that the horizon moon appears to be closer than the zenith moon, return to Equation 2 and assume  $d'_h = d'_z$  and  $s'_h > s'_z$ . By substitution in Equation 2,

<sup>1</sup> We also ignore scaling constants that may be necessary in comparing different units of measure.

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we can conclude first that  $\theta'_h > \theta'_z$ . Next, rearrange the basic equation again so that

$$d' = \frac{s'}{\theta'} \quad (4)$$

Now, by assuming  $s'_h = s'_z$  and  $\theta'_h > \theta'_z$ , we conclude that  $d'_h < d'_z$  (Fact 2). The remarkable feature of this sequence of reasoning is that contradictory assumptions are made about every term in the equations (perceived size, perceived distance, and perceived visual angle).

ceived size, perceived distance, and perceived visual angle).

Rock and Kaufman (1962) attempted to remedy matters by assuming two types of perceived distance: unconsciously registered distance to explain Fact 1, and consciously judged distance to explain Fact 2. This modification raises more unanswered questions (e.g., Restle, 1970) without even addressing the contradictions arising from the disparate assumptions made about perceived size and perceived visual angle.

Most recent theories have attempted to circumvent these logical contradictions. McCready (1986) claims that what we should mean by the apparent size of the moon is its perceived visual angle as opposed to its perceived metric size. The advantage of this notion is that the moon can appear both smaller and farther away, or larger and nearer, without producing internal contradictions in Equation 2; that is, Facts 1 and 2 are reconciled. Presumably, the perceived visual angle diminishes with increased elevation of the moon either because of the functioning of ocular variables (e.g., accommodation) or because of associated central, efferent commands. We feel these variables are of minor importance at the great distances involved in the moon illusion (Wagner, Baird, & Fuld, 1989). Efference, as an explanatory concept, is also quite difficult to validate.

Building on Restle's earlier work (1970), we previously developed a reference theory of the moon illusion (Baird, 1982). The key claim was that the apparent size and distance of the moon, as well as all other visual objects, depend on the presence of the visual context provided by other stimulus objects in the field of view. Essentially, if the perceived size of the moon is compared to large referent objects, it will appear small, and vice versa. One possible important referent is the sky. When the moon is seen at the zenith, it is surrounded by a large expanse of open sky, and therefore, it appears small. When it is close to the horizon, it is seen in the vicinity of smaller visual extents (including only half the sky), and therefore, it appears large. There are two major drawbacks with this *sky model*. First, it is not clear whether apparent size refers to metric or angular extents. Second, the sky model predicts no difference in apparent size as a function of apparent distance to the horizon, so long as the proportion of sky seen by the observer is unchanged. This is contradicted by Fact 4. For example, as Bishop George Berkeley (1709; cited in Turbayne, 1963) noted, the size illusion disappears when the moon is seen over a nearby wall; Rock and Kaufman (1962) showed that the moon illusion is reduced when the perceived distance to the horizon is reduced.

## Our Proposal

The point of this report is to show that a combination of the sky model (Baird, 1982), parts of McCready's theory (1986), and one well-known perceptual phenomenon successfully addresses the facts associated with the moon illusion, as well as suggesting plausible explanations of secondary observations related to the illusion. Overall, this combination can be seen as an outgrowth of Restle's approach (1970).

The three principles driving this new explanation are as follows. Principle 1: By apparent size of objects, we mean their perceived visual angle. Principle 2: The apparent size of the moon is determined by the ratio of the angular extent of the moon relative to the extents subtended by objects composing the surrounding context. Principle 3: The visual extents subtended by common objects of a constant physical size decrease systematically with increasing distance from the observer. Perfect examples of this are Gibson's (1950) texture gradients, which are reproduced in virtually every elementary textbook on perception.<sup>2</sup>

The quantitative model we use is of the psychophysical variety (Baird, 1970; Stevens, 1975):

$$\theta' = k \left( \frac{\theta}{\xi_r} \right)^n, \quad (5)$$

where the perceived size of the moon ( $\theta'$ ) is a power function of the ratio of the visual angle of the moon to the visual angle of a referent ( $\xi_r$ ), which may represent a weighted average over a number of different visual extents. Because we are dealing with linear extent, we assume that  $n = 1$  and that  $k$  is a scaling constant whose purpose is to convert units of measure from stimulus ratios into perceived visual angle<sup>3</sup> (Baird, 1970). We thus assume the scaling constant to be such under all conditions for viewing the moon. It is commonplace to discuss the moon illusion in terms of the perceived size at the horizon in respect to the perceived size at the zenith or some other elevation. Therefore, according to Equation 5, we end up with a dimensionless ratio of the type:

$$\frac{\theta'_h}{\theta'_z} = \frac{\xi_z}{\xi_h}. \quad (6)$$

<sup>2</sup> We have shown that observers walking through a natural environment attend to the same objects at different distances (Wagner, Baird, & Barberesi, 1981). Principle 3 is alluded to by Restle (1970) and is also mentioned by Solhkah and Orbach (1969) as a possible explanation of the enlarged size of the moon at the horizon (Solhkah & Orbach, 1969, Figure 3, p. 91). The latter authors do not use Principles 1 and 2, and apparently favor a model based on perceived metric extent (Solhkah & Orbach, 1969, p. 94).

<sup>3</sup> The reason that relative size affects perceived visual angle in this manner is not at all clear. One possibility is that the observer attends primarily to that portion of the visual field occupied by the largest referent (e.g., the sky), and hence, other objects appear large or small with respect to this maximum field of attention.

This derived ratio offers a convenient representation, but it is not a substitute for Equation 5, which is the appropriate expression for the perceived visual angle of the moon.

Our three principles explain the major facts in the following manner. Fact 1: When the moon is seen at a high elevation, its visual angle is compared against the wide expanse of the sky. Therefore, in Equation 5,  $\xi_z$  is relatively large and the moon ( $\theta'_z$ ) appears small. When the moon is seen at the horizon, it is compared against the smaller visual extents of objects at the distant horizon, and therefore,  $\xi_h$  is relatively small and the moon ( $\theta'_h$ ) appears large. A weighting function over the stimulus referents will have to be worked out in order to describe the systematic change in size with changes in elevation between the horizon and the zenith (cf. the sky model [Baird, 1982] with Restle's [1970] adaptation level model). Fact 2: Because the perceived extent of the moon is relatively smaller at the zenith than it is at the horizon, and because the observer assumes the moon is of fixed physical size ( $s'_h = s'_z$ ), the moon is seen as closer at the horizon than at the zenith, in accord with Principle 3. That is, by substituting in Equation 4 and taking ratios,

$$\frac{d'_h}{d'_z} = \frac{\theta'_z}{\theta'_h}. \quad (7)$$

It should be noted that the explanations of Facts 1 and 2 do not introduce any contradictions among the terms in Equations 4–7. Fact 3: If the visual terrain is eliminated or the moon is viewed through an occluding tube, the moon is compared to a large homogeneous expanse, much as it is when seen against the zenith sky. Therefore,  $\xi_z$  is large, and the moon appears small. Fact 4: By Principle 3, as the distance to the horizon increases, the visual angles subtended by objects of the same physical size decrease (Equation 1). The moon at the horizon is therefore compared against smaller and smaller visual extents ( $\xi_h$  becomes progressively smaller) as distance to the horizon increases. This results in the moon's being seen as larger at greater distances, even though it is the comparison against smaller visual extents that causes the illusion and not the perceived distance to the horizon. This would occur for any surface that provides ample visual texture cues, such as over a field, a desert, a lake, or an ocean.

Subsidiary observations are handled as follows. Observations 1 and 2: If the moon at the horizon is framed by either buildings or clouds, it will be seen as larger because the visual angle of the enclosing frame is relatively small. Alternatively, puffy clouds or the windows in buildings may provide small details against which the moon is seen as relatively large. Clouds in the sky overhead may also be physically closer, thus providing a larger frame of reference than they do at the horizon. Observation 3: Inverting the terrain disrupts the ability to recognize the same objects at different distances (Rock, 1983, p. 48), and, hence, disrupts the usual tendency to perceive smaller visual extents as distance to the horizon increases. With the scene inverted, the average value of the

referent ( $\xi_r$ ) in Equation 5 increases, because smaller visual extents are omitted from the composite through lack of object recognition and visual attention. It is not generally recognized by theorists that Equation 1 only has perceptual consequences if observers attend to the same object at different distances. The limiting counterexample would be a situation in which there is no realization that the same objects are present at different distances, and therefore, the referent ( $\xi_r$ ) remains the same. Under these circumstances, the apparent size of the moon would not change when viewed above different extents of terrain.

Our coordination of the three simple perceptual principles accounts for the major facts and observations associated with the moon illusion. We know of no other theory of this phenomenon that explains so much with so little.

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